Matrices

Matrix

A matrix is an ordered rectangular array of numbers or functions, such an array is enclosed by [ ] or ( ). The numbers or functions are called the Elements or the entries of a matrix.

A matrix is represented by capital letters like A, B, C... and elements of a matrix is represented by small letters like a, b, c...

In a matrix, horizontal lines are called Rows and vertical lines are called Columns.

eg: $$A = \begin{array}{cc} \left[\begin{array}{cc} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{array}\right] & \begin{array}{l} \to 1^{st} \text{ row } (R_1) \\ \to 2^{nd} \text{ row } (R_2) \\ \to 3^{rd} \text{ row } (R_3) \end{array} \\ \begin{array}{cc} \downarrow & \downarrow \\ 1^{st} \text{ Column } (C_1) & 2^{nd} \text{ Column } (C_2) \end{array} & \end{array}$$

Order of Matrix

Suppose a matrix has m rows and n Columns Then order of matrix is written as $m \times n$, where m represents the number of rows and n represents the number of Columns. It is read as m by n

In general, a $m \times n$ matrix can be represented as

$$a_{ij} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

a_{ij} is an element lying in the i^{th} row and j^{th} Column.

The number of elements in $m \times n$ matrix is equal to mn eg: Consider the matrix $A = \begin{bmatrix} 2 & 3 & 3/2 & 1 \\ 6 & 8 & 9 & 7 \\ 3 & 4 & 5 & 10 \end{bmatrix}$

a) Write the order of this matrix

b) Write the values of the elements at

$a_{13}, a_{21}, a_{33}, a_{24}$

Sol: a) The order of a matrix is given by $m \times n$

Here, there are 3 rows and 4 columns.

Therefore, order of the matrix $A = 3 \times 4$

b) Here a_{13} represents the element at

1st row and 3rd column.

$\therefore \; a_{13} = 3/2$

Similarly, $a_{21} = 6$, $a_{33} = 5$, $a_{24} = 7$

Q: Construct a matrix of order 2×2 whose

elements are determined by $a_{ij} = i + j$

Sol: The matrix A can be represented as

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$a_{11} = 1+1 = \underline{\underline{2}}$

$a_{12} = 1+2 = 3$

$a_{21} = 2+1 = 3$

$a_{22} = 2+2 = 4.$

$$\therefore A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}_{2\times 2}$$

Q. Construct a matrix of order 3×2 whose elements are determined by $a_{ij} = \dfrac{2i-j}{3}$.

Sol. $A = a_{ij} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

$a_{11} = \dfrac{2\times1-1}{3} = \underline{\underline{1/3}}$, $a_{12} = \dfrac{2\times1-2}{3} = \underline{\underline{0}}$

$a_{21} = \dfrac{2\times2-1}{3} = \underline{\underline{1}}$, $a_{22} = \dfrac{2\times2-2}{3} = \underline{\underline{2/3}}$

$a_{31} = \dfrac{2\times3-1}{3} = \underline{\underline{5/3}}$, $a_{32} = \dfrac{2\times3-2}{3} = \underline{\underline{4/3}}$

$$\therefore A = \begin{bmatrix} 1/3 & 0 \\ 1 & 2/3 \\ 5/3 & 4/3 \end{bmatrix}_{3\times 2}$$

Types of Matrices

1. Row Matrix

A matrix having only one row is called Row matrix

eg: $A = \begin{bmatrix} a & b & c \end{bmatrix}$

2. Column Matrix

A matrix having only one column is called Column matrix eg: $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

3. Square Matrix

A matrix in which no. of rows and no. of columns are equal.

eg. $A = \begin{bmatrix} 1 & 2 \\ 7 & 9 \end{bmatrix}$

4. Diagonal Matrix

A square matrix A is said to be a diagonal matrix if all elements lying outside the diagonal elements are zero.

eg. $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

5. Scalar Matrix

A diagonal matrix in which all diagonal elements are equal.

eg: $A = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$

6. Unit Matrix / Identity Matrix

A square matrix having 1 on its principle diagonal and zero elsewhere is called an Identity Matrix.

eg: $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Identity Matrix of order 2×2

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, order 3×3

7. Zero Matrix / Null matrix

If all the elements of a matrix is zero then it is a Zero Matrix.

$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Q. Identify the following types of matrices

1. $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & 4 \\ 4 & 6 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Sol: 1 → Scalar Matrix

2 → Square Matrix

3 → Diagonal Matrix

4 → Column Matrix

Equality of Matrix

Two matrices A and B are said to be equal if their order are same and their corresponding elements are equal.

eg: $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

Here both matrices A and B are of order 2×2 and their elements are equal. Hence $A = B$

Q. Find the Values of a, b, c when

$$\begin{bmatrix} a & 3 \\ 2 & c \end{bmatrix} = \begin{bmatrix} 0 & b \\ 2 & 1 \end{bmatrix}$$

Sol: Let $A = \begin{bmatrix} a & 3 \\ 2 & c \end{bmatrix}$, $B = \begin{bmatrix} 0 & b \\ 2 & 1 \end{bmatrix}$

Given, $A = B$

$\therefore a = 0, \; b = 3, \; c = 1$

* Find x, y, z and w, $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Sol: $x - y = -1$ —— (1)

$2x - y = 0$ —— (2)

Substracting eq. (1) from (2) we have,
i.e, (2) − (1)

$$\begin{array}{r} 2x - y = 0 \\ -x + y = 1 \\ \hline x = 1 \end{array}$$

Substituting the value of x in (1)

$1 - y = -1 \Rightarrow \underline{\underline{y = 2}}$.

Similarly, we have,

$$2x + z = 5$$

We have $x = 1$,

Substituting we get,

$$2 \times 1 + z = 5$$

$$\therefore z = 5 - 2 = \underline{\underline{3}}$$

Also,

$$3z + w = 13$$

Substituting the value of z we have,

$$3 \times 3 + w = 13 \Rightarrow w = 13 - 9 = \underline{\underline{4}}$$

$\therefore$ $x = 1$, $y = 2$, $z = 3$ and $w = 4$